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Adjoint-Based Aerodynamic Shape Optimisation for Three-Dimensional Hypersonic Configurations

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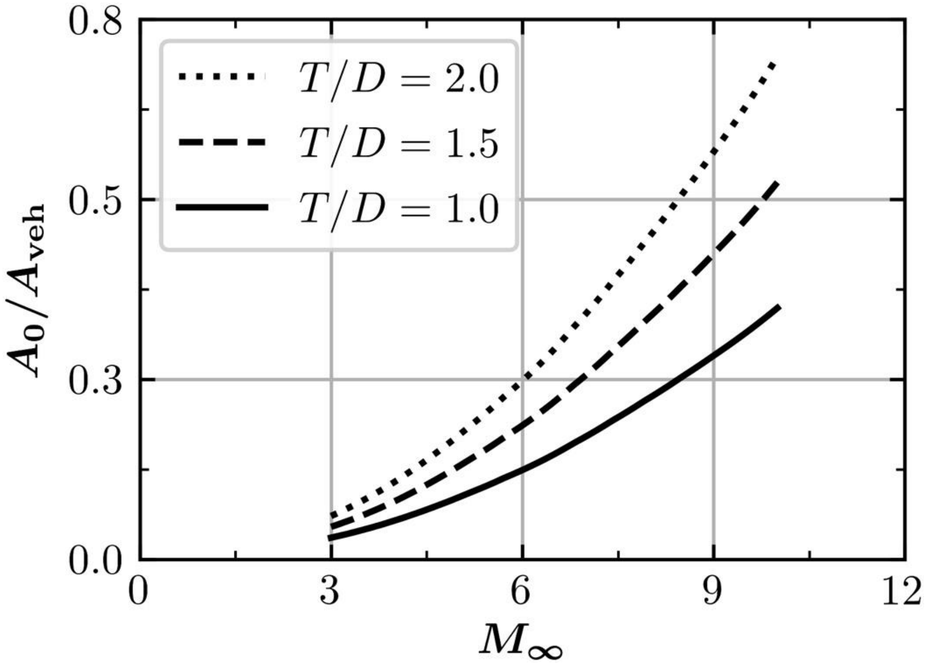
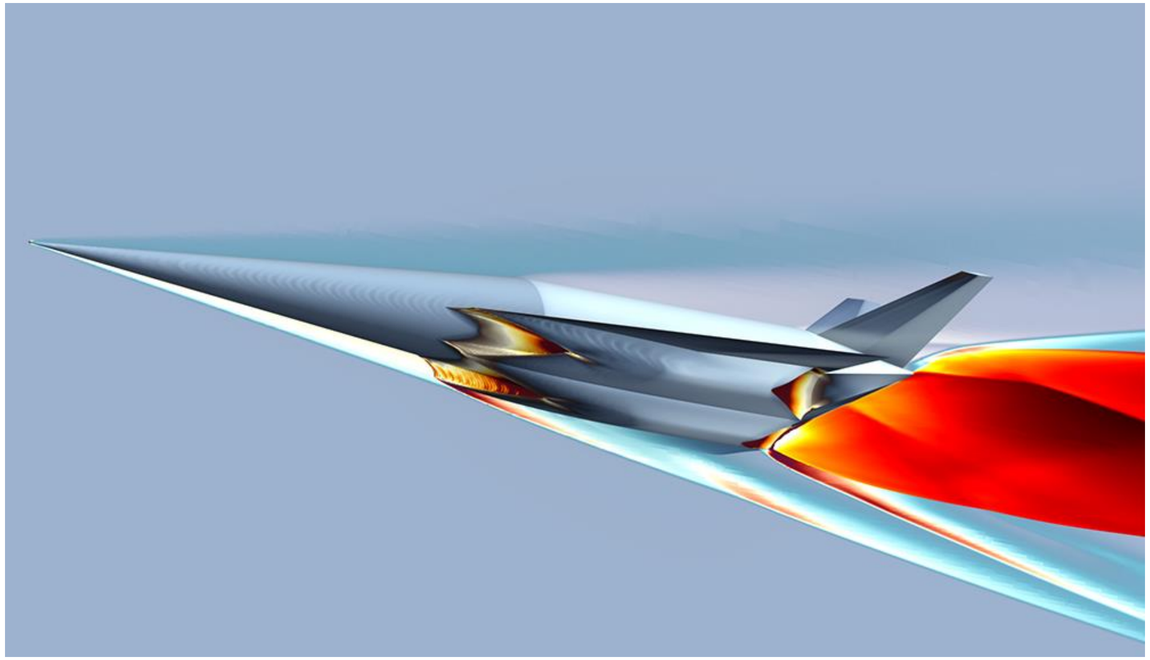
Hypersonic Airbreathing Accelerator Design



Relative capture area vs Mach number (Ward & Smart, 2021)

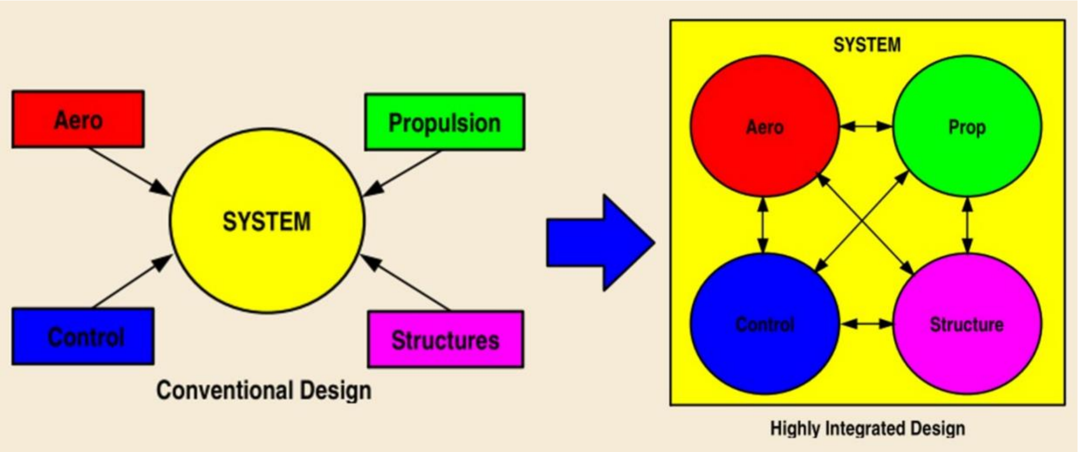


Delta-Velos by Hypersonix Launch Systems (Hypersonix, 2021).

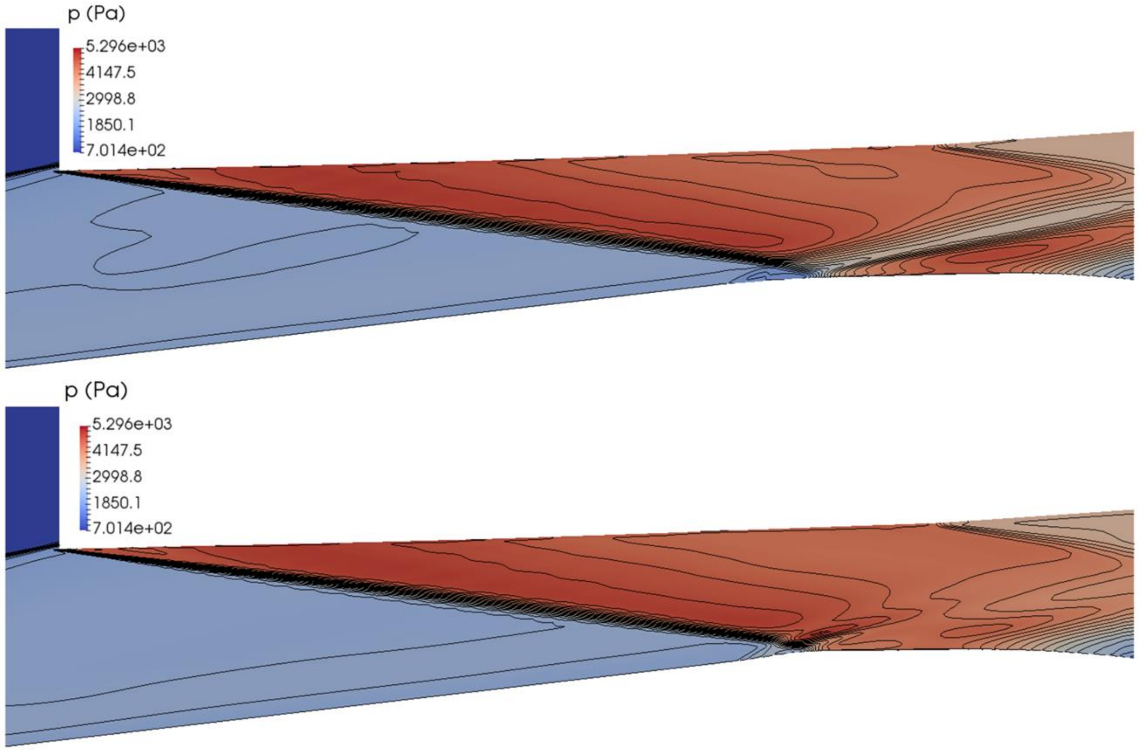
- Required engine size increases with M_∞ for airbreathers
- Integrate propulsion system with airframe

- Strong coupling between propulsion and airframe
- Many components serve multiple roles with competing demands
- Non-intuitive design trade-offs

Hypersonic Design Optimisation



Conventional vs coupled design (Bowcutt, 2003).



Minimisation of inlet pressure distortion (Damm, 2019).

- Multidisciplinary Design Optimisation (MDO) for coupled, system-level design
- Many solvers and design parameters
- Must limit fidelity & design space

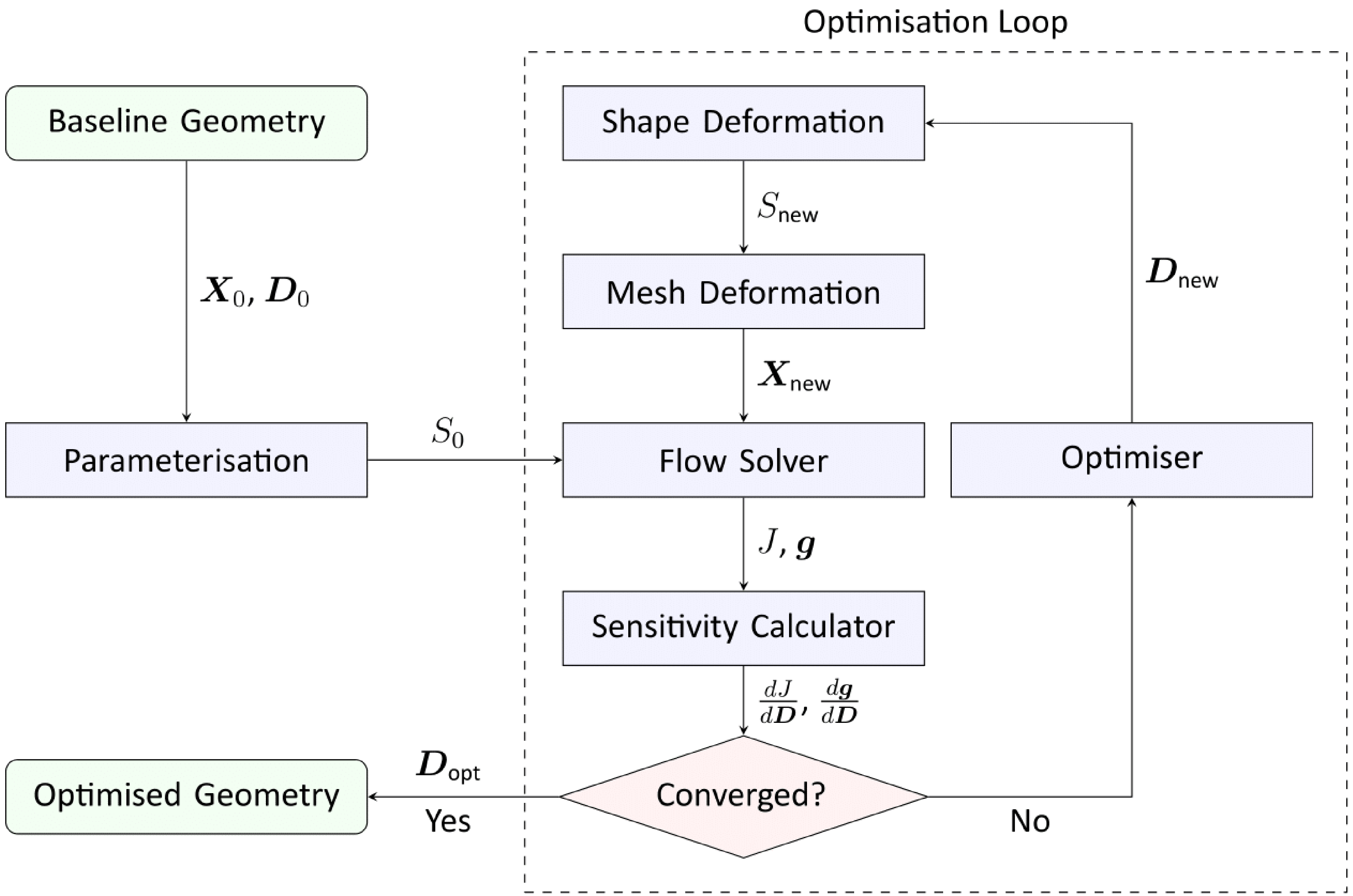
- Aerodynamic Shape Optimisation (ASO) with CFD
- Efficient many-parameter optimisation with the adjoint method
- Mainly 2D geometries in literature

Aim and Talk Outline

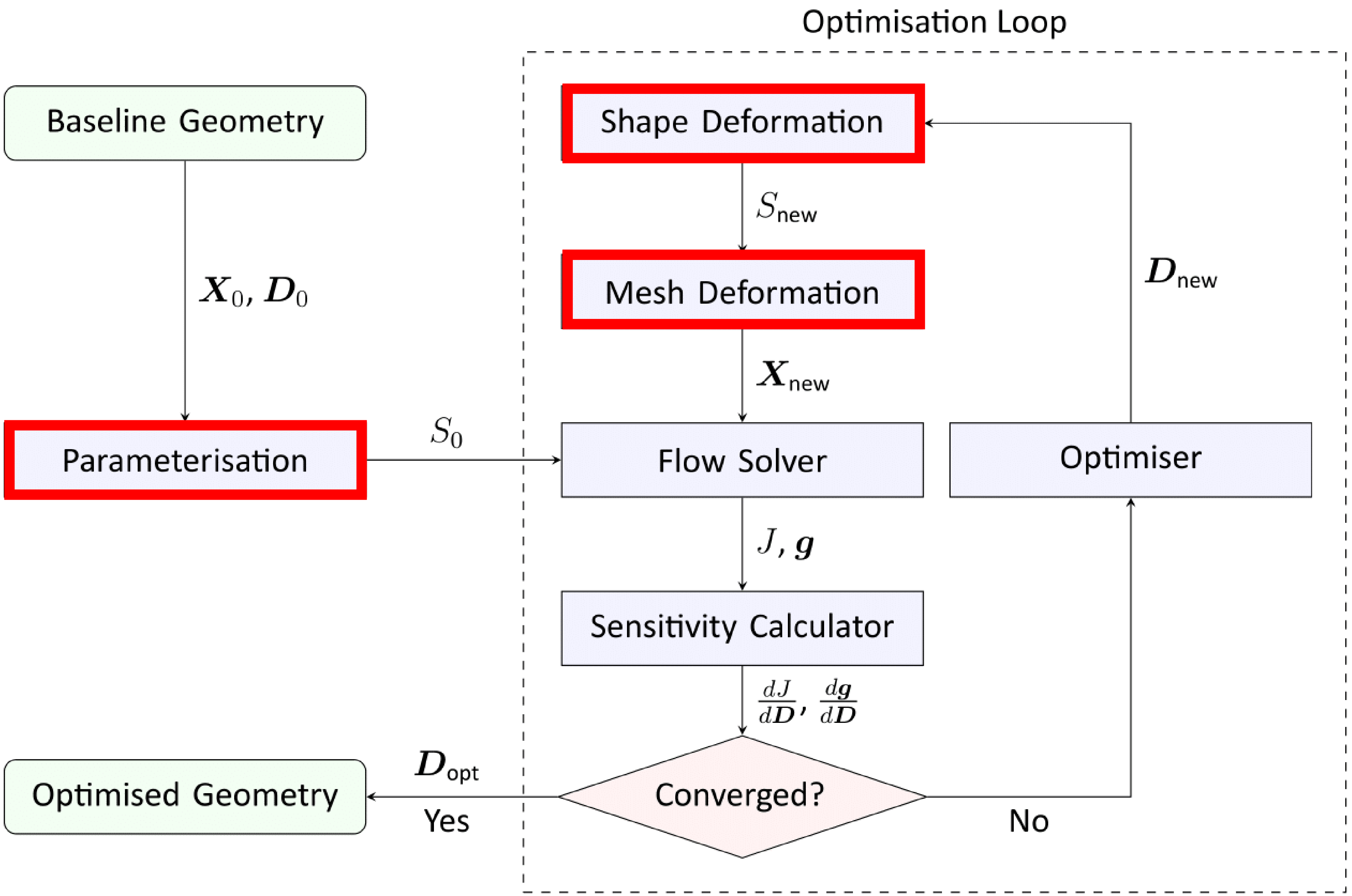
Develop an adjoint-based ASO framework for efficient design of 3D hypersonic configurations

1. Aerodynamic shape optimisation methodology
   - Detail capability and each core sub-routine
2. Aerodynamic shape optimisation of a hypersonic lifting body
   - Demonstration of capability for a complex design task
3. Conclusions

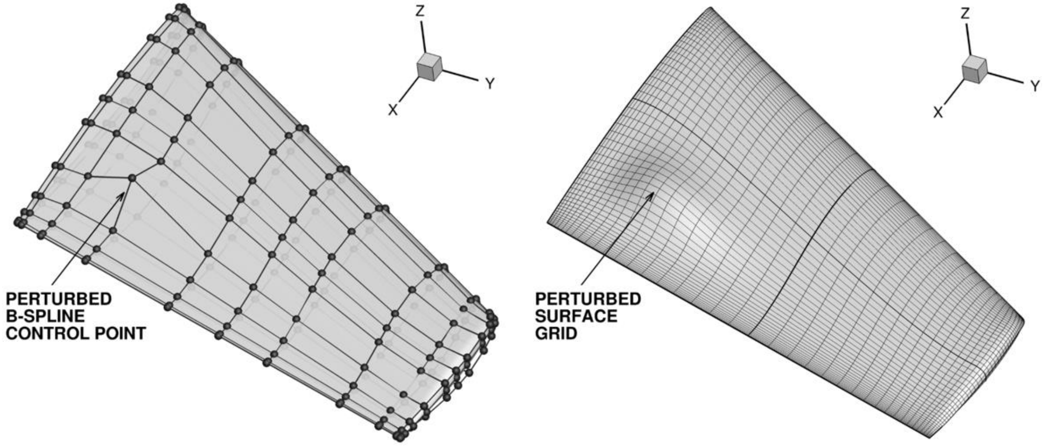
ASO Methodology – Optimisation Routine



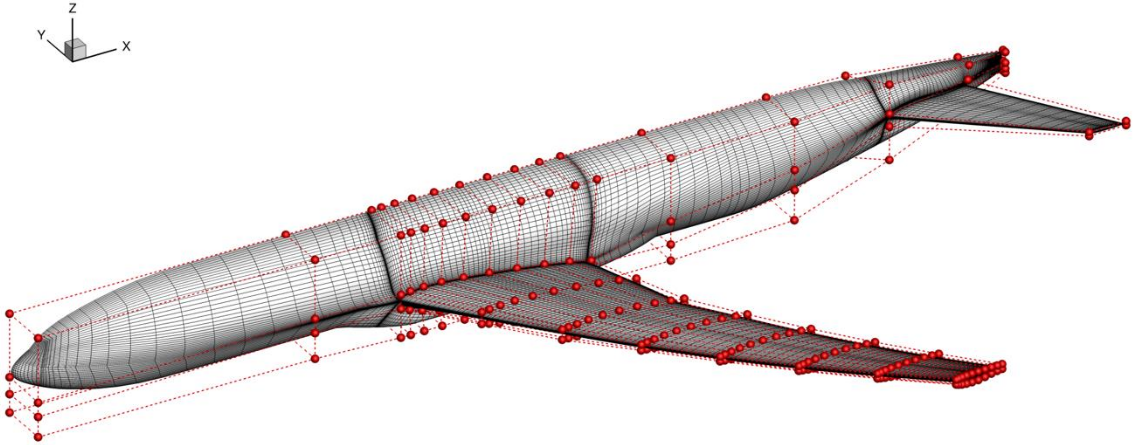
ASO Methodology – Geometric Parameterisation & Deformation



Geometric Parameterisation Methods



B-spline wing surfaces (Leung & Zingg, 2012).



Free-form deformation of aircraft (Kenway, 2013).

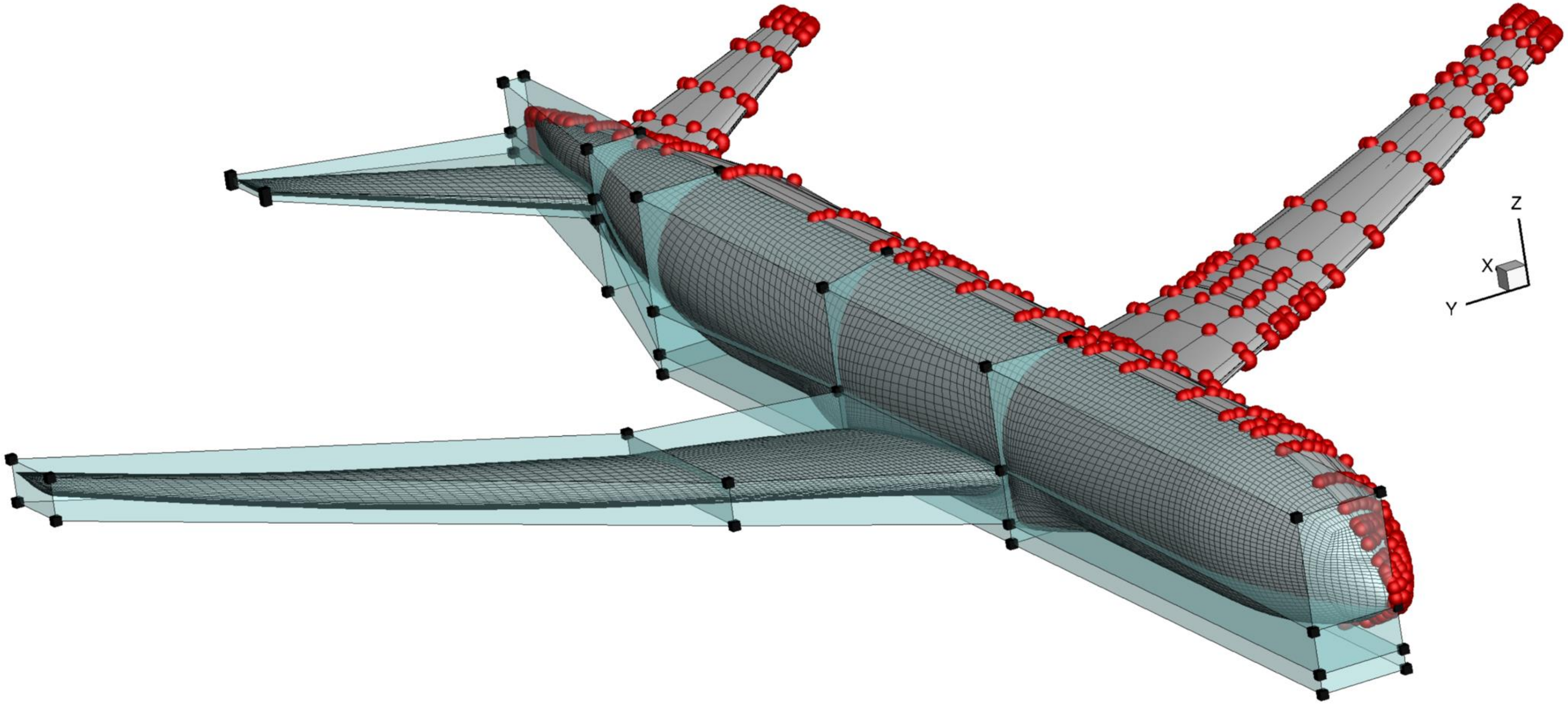
B-spline/NURBS surfaces

- High local control, analytical
- Integrated mesh deformation
- Susceptible to overlap, many DVs

Free-form deformation (FFD)

- General, robust, easy DV selection
- Discrete surface representation
- Separate grid deformation method

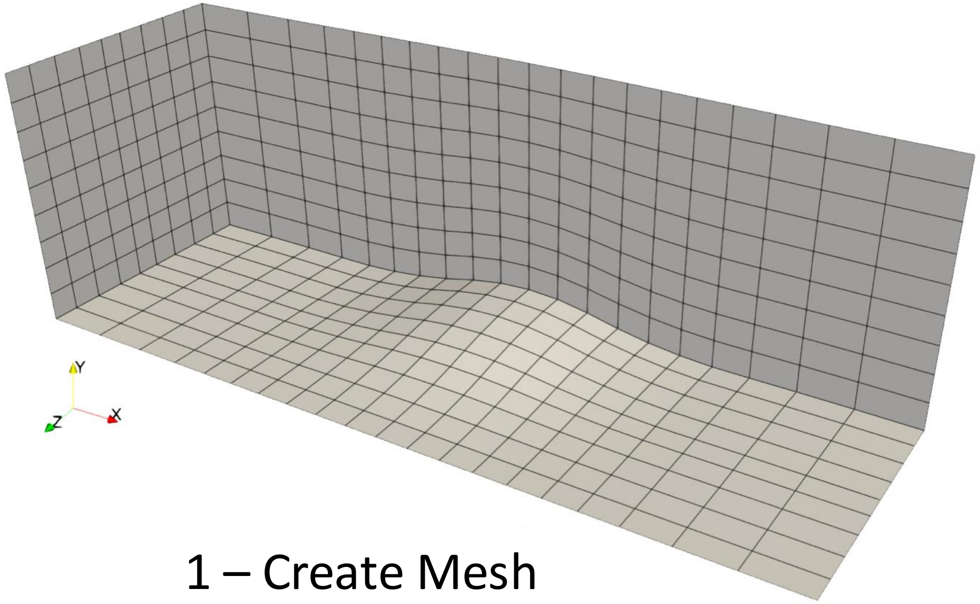
Two-Level Free-Form Deformation



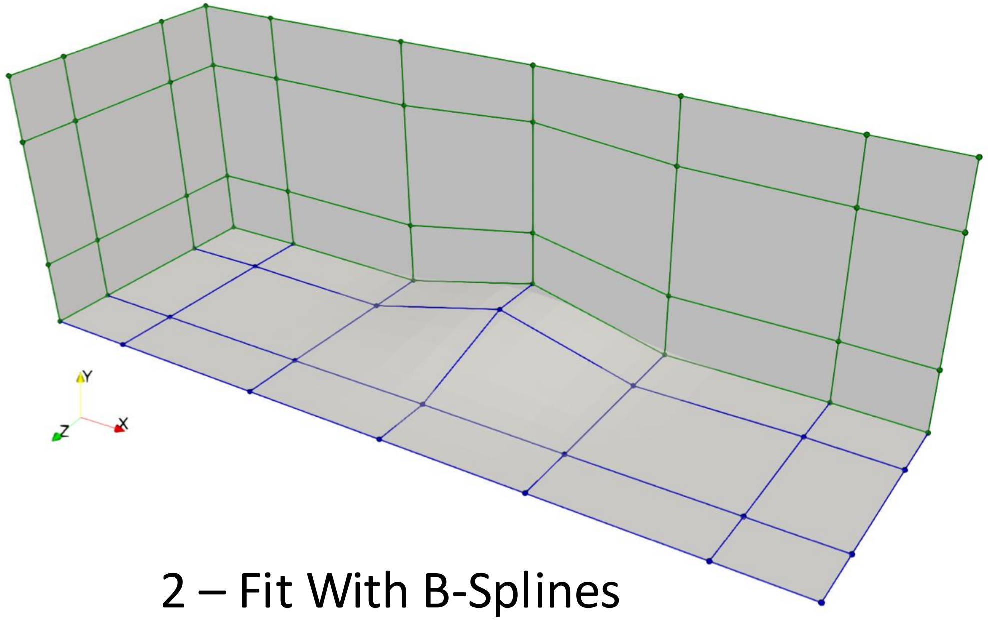
FFD parameterisation (left) and B-spline parameterisation (right) (Kenway, et al., 2010).

- Embed B-spline surfaces in FFD hulls – two-level FFD by Gagnon & Zingg, 2015
- Inherit advantages of both methods
- Use B-spline volumes to deform volume mesh

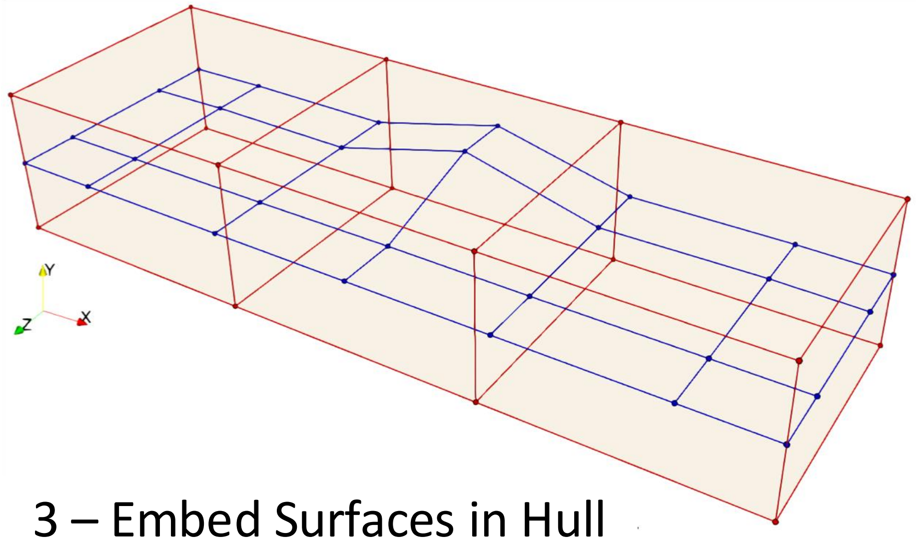
Two-Level Free-Form Deformation – Parameterisation Methodology



1 – Create Mesh

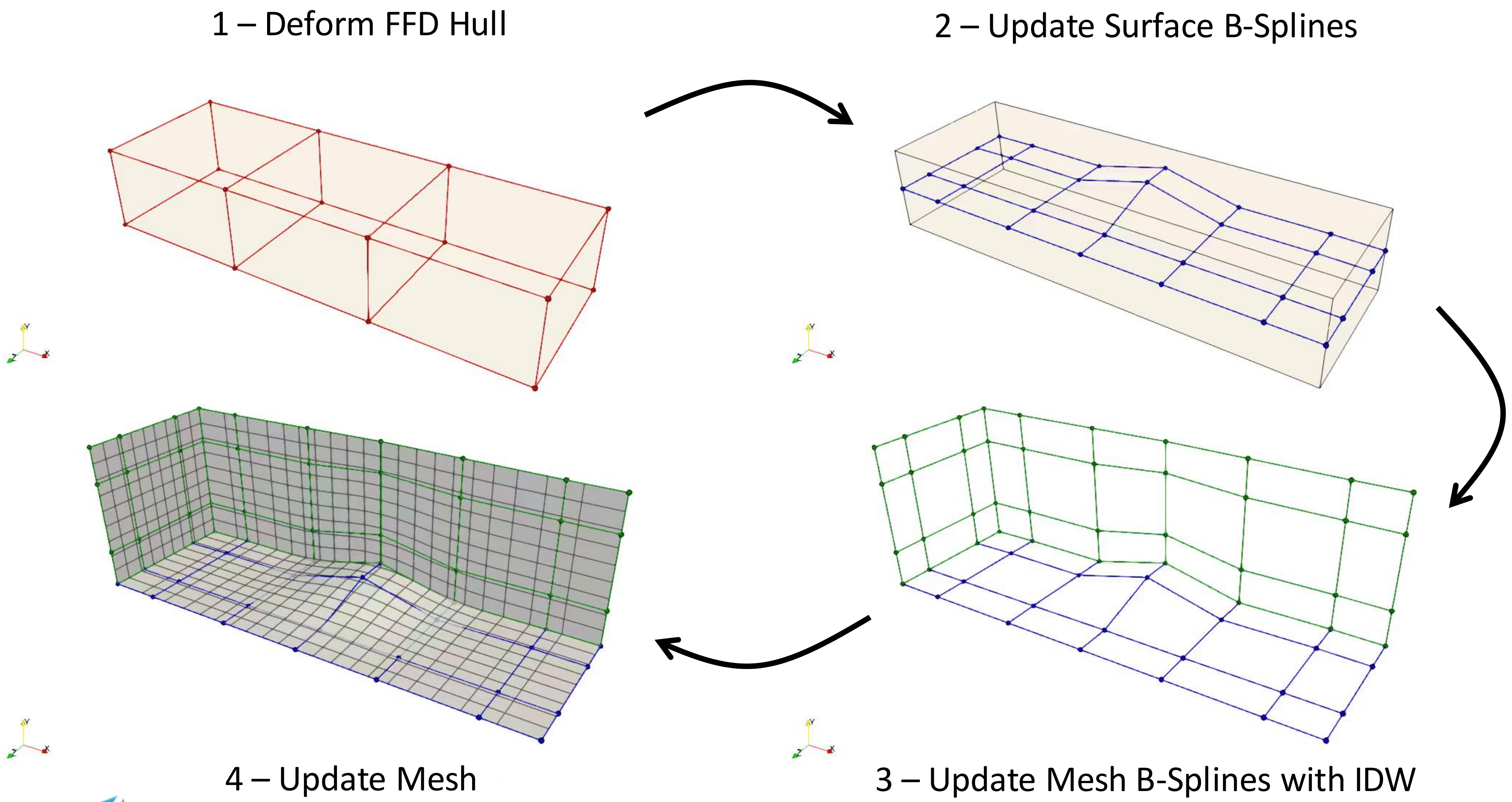


2 – Fit With B-Splines

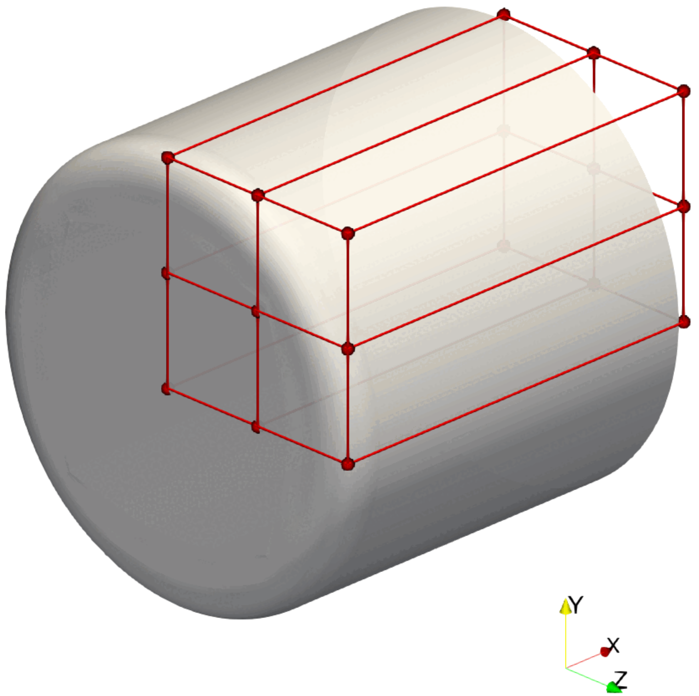


3 – Embed Surfaces in Hull

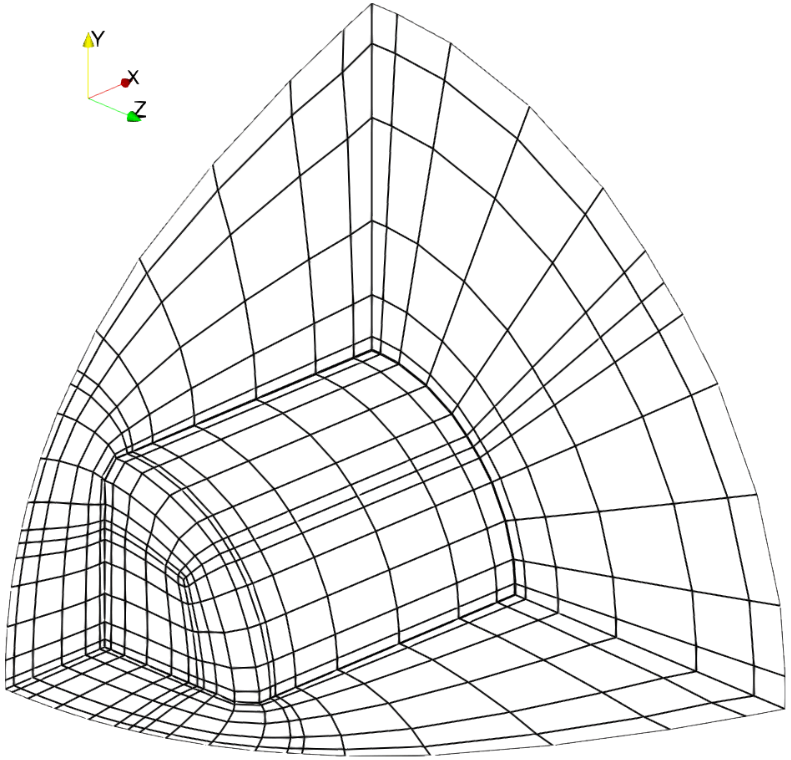
Two-Level Free-Form Deformation – Deformation Methodology



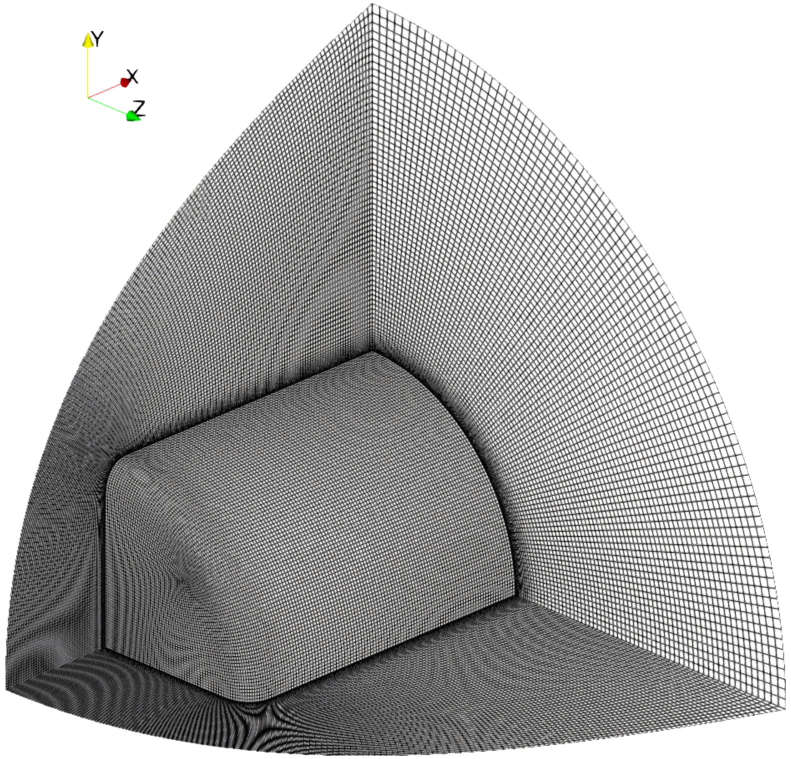
Two-Level Free-Form Deformation – Cylinder to Re-Entry Vehicle



Surface & FFD hull

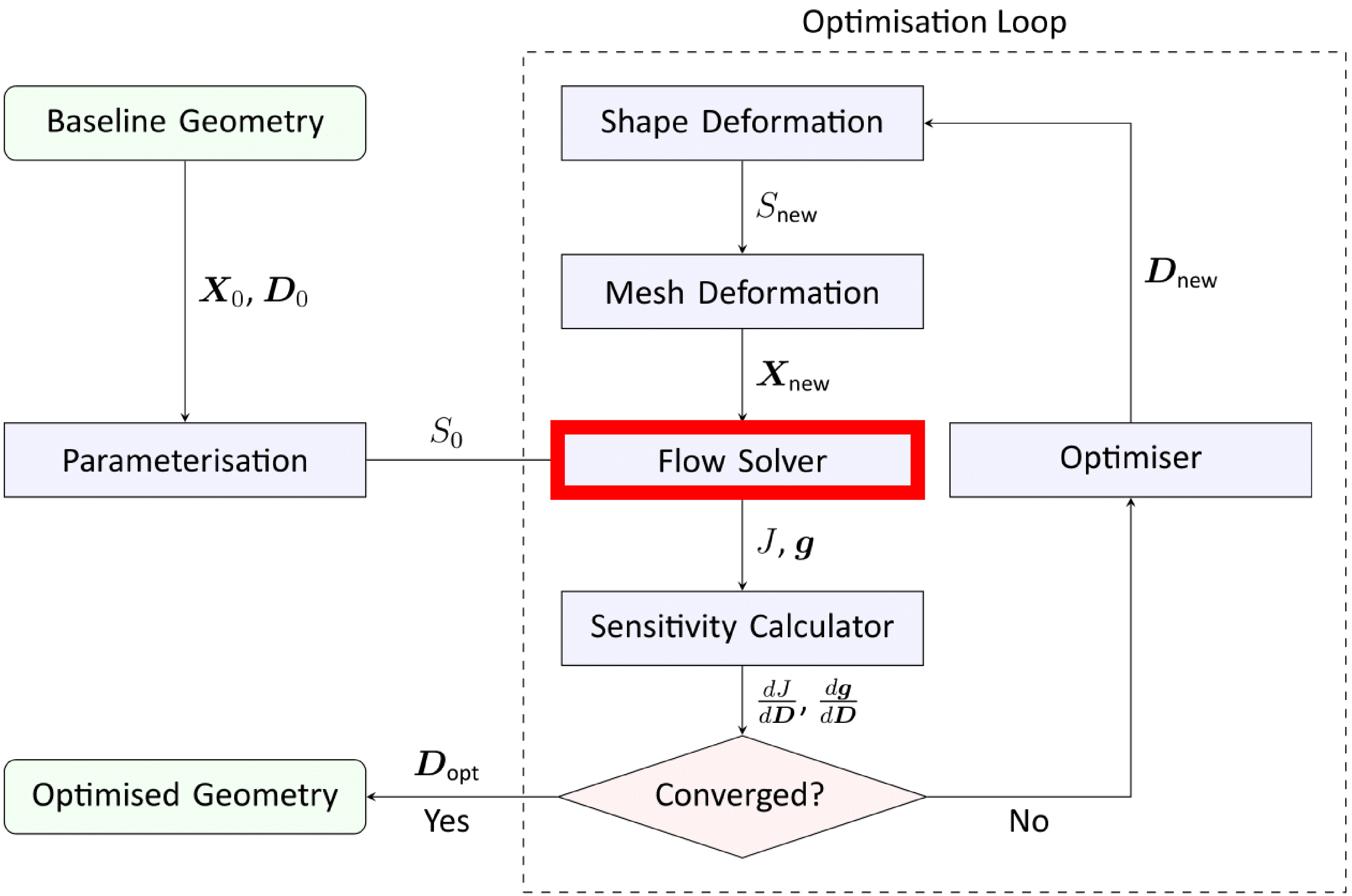


Surface & Mesh B-Splines



Computational Mesh

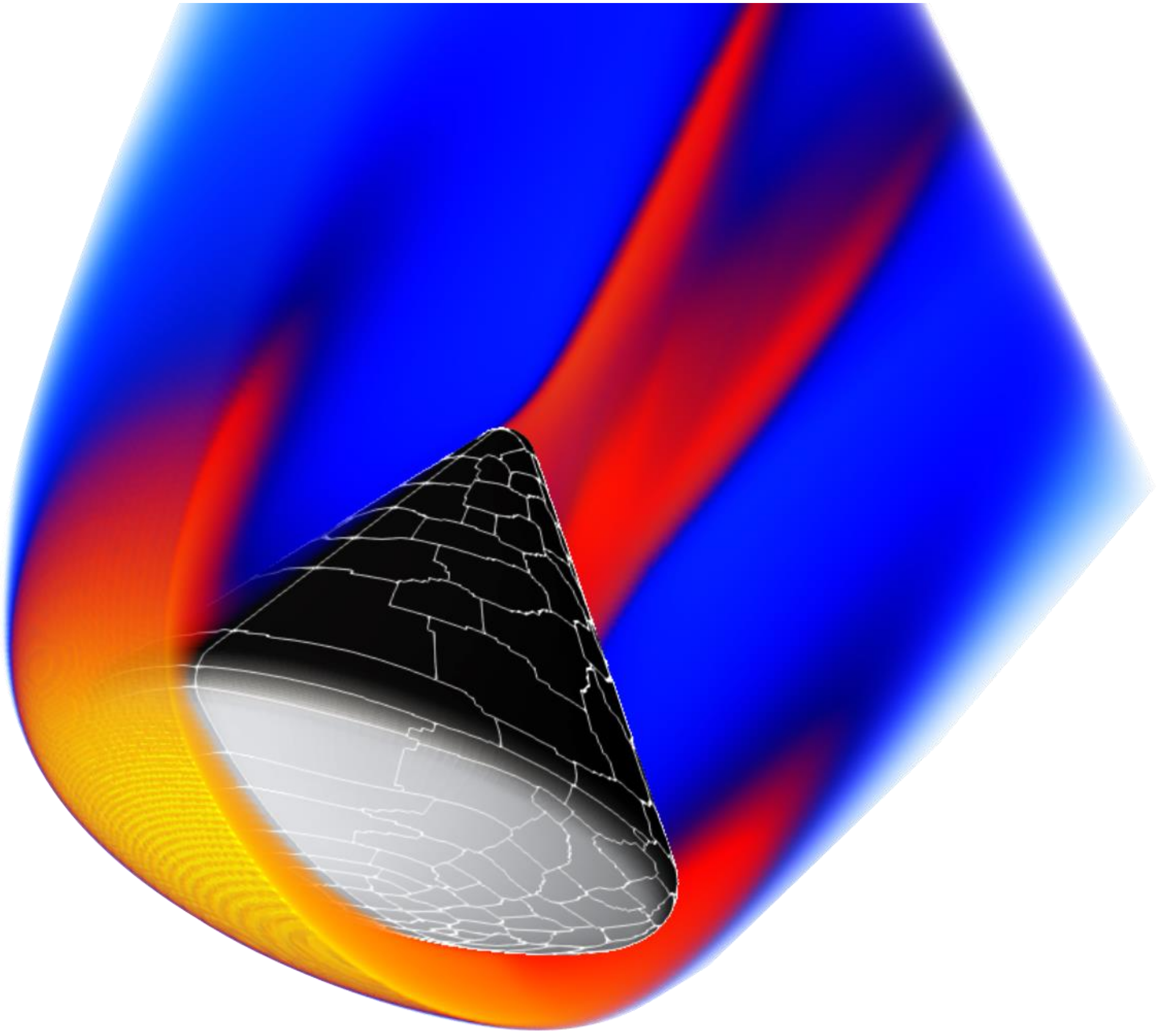
ASO Methodology – Flow Solver



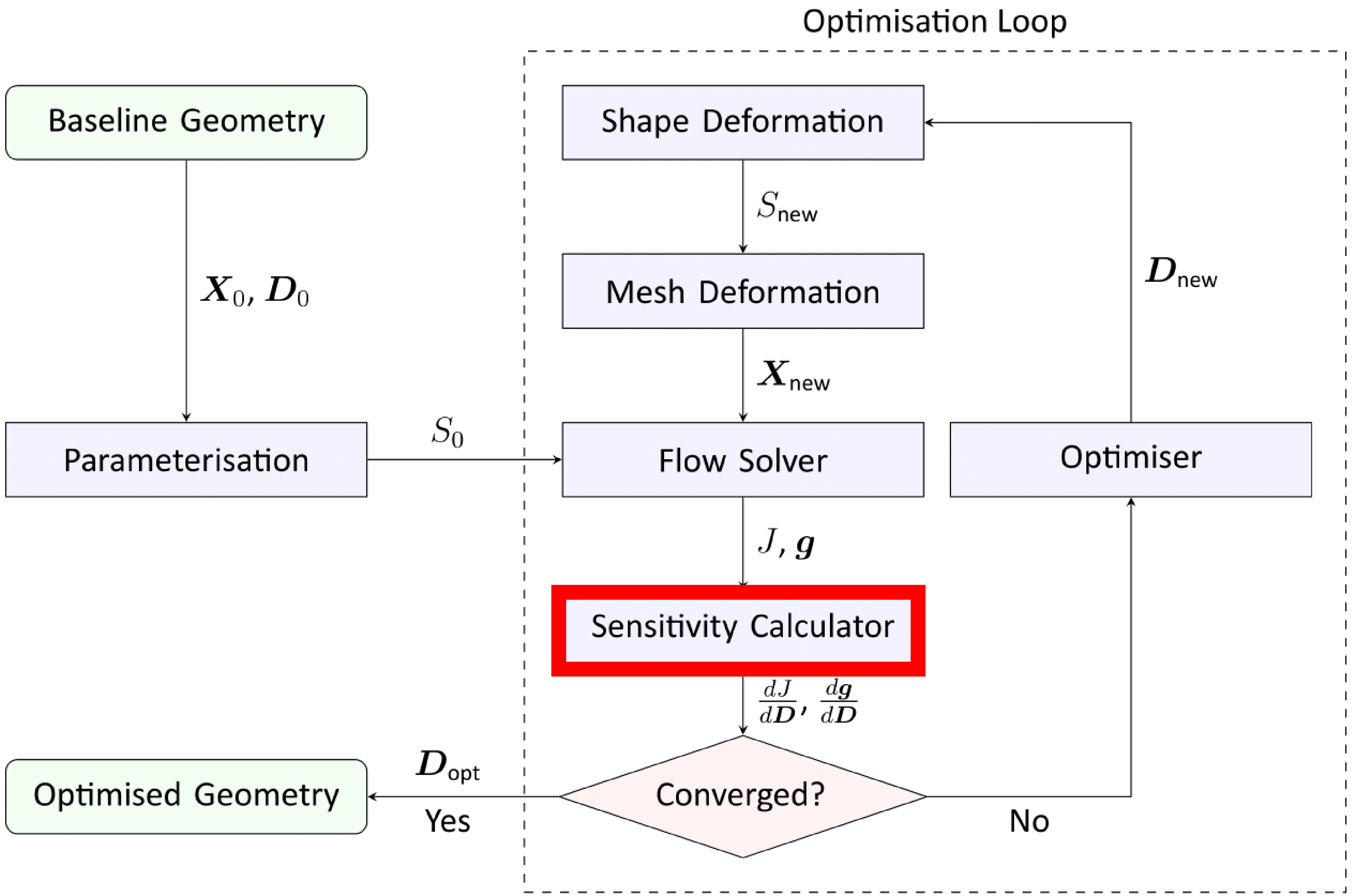
Flow Solver – Eilmer

Eilmer
- Open-source hypersonic flow solver from UQ
- Thermochemical non-equilibrium & turbulence modelling
- Transient & steady-state solvers
- 2D & 3D structured/unstructured grids

Current capabilities for adjoint-based ASO
- Jacobian-Free Newton-Krylov (JFNK) solver
  - Steady-state – deep convergence
- Euler, Navier-Stokes, RANS
- Unstructured grids
- Ideal air gas model


Eilmer simulation of the Apollo capsule (Gibbons, 2024)

ASO Methodology – Sensitivity Calculator



Shape Sensitivity Calculation – The Adjoint Method

Objective/constraint function f depends on the grid *($\boldsymbol{X}$)*, flow state *($\boldsymbol{U}$)* and design vars *($\boldsymbol{D}$):*

$$f = f\big(\boldsymbol{X}(\boldsymbol{D}), \boldsymbol{U}(\boldsymbol{D})\big)$$

$$\frac{df}{d\boldsymbol{D}} = \frac{\partial f}{\partial \boldsymbol{X}}\frac{d\boldsymbol{X}}{d\boldsymbol{D}} + \frac{\partial f}{\partial \boldsymbol{U}}\frac{d\boldsymbol{U}}{d\boldsymbol{D}}$$

<u>Finite differences</u>

Requires $N + 1$ flow solutions for N design variables

<u>Adjoint method</u>

By assuming steady flow ($\boldsymbol{R} = \partial \boldsymbol{U}/\partial t = \boldsymbol{0}$) & employing Lagrange multipliers:

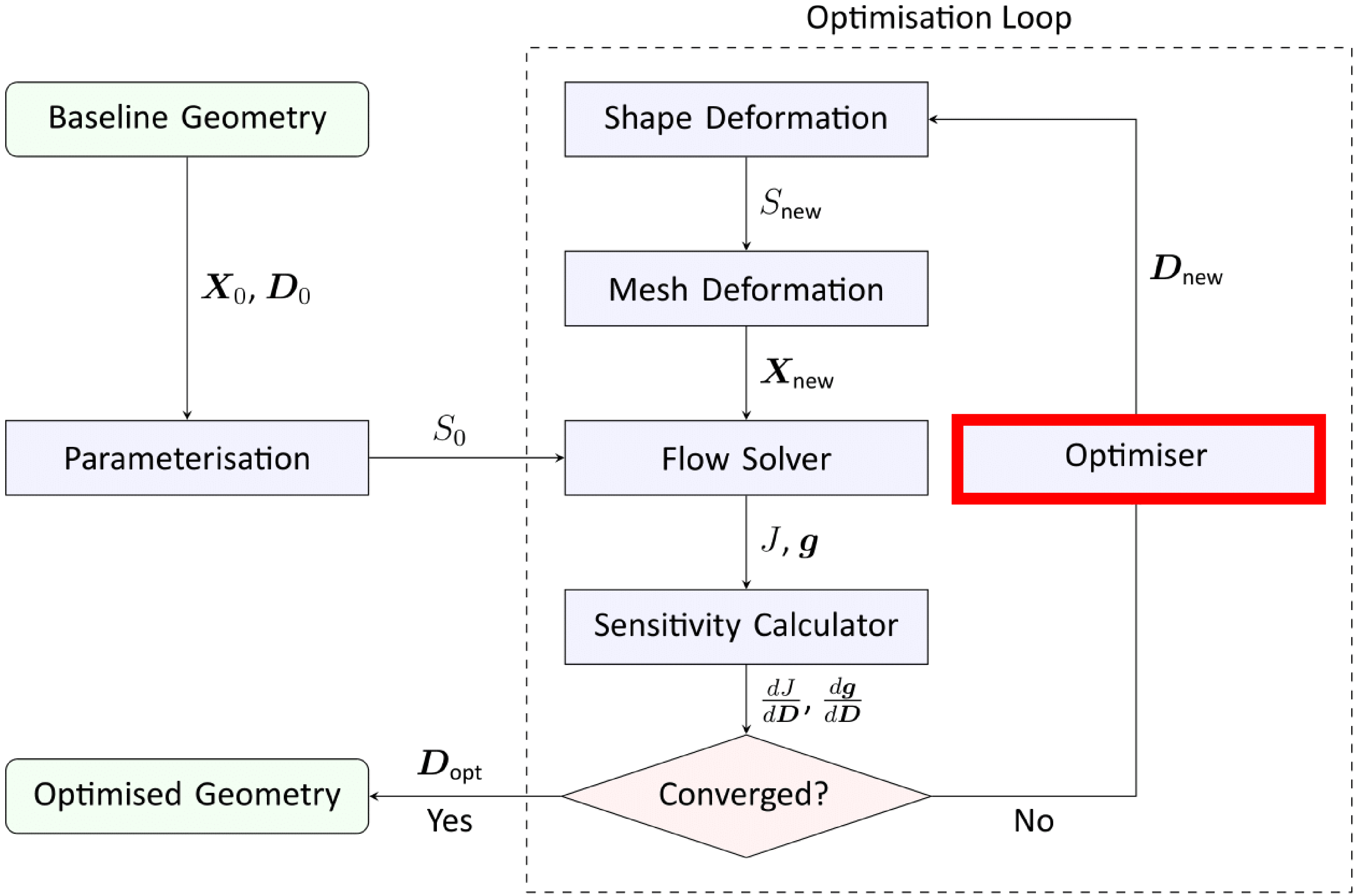
$$\frac{df}{d\boldsymbol{D}} = \frac{\partial f}{\partial \boldsymbol{X}}\frac{d\boldsymbol{X}}{d\boldsymbol{D}} + \boldsymbol{\lambda}^T \frac{\partial \boldsymbol{R}}{\partial \boldsymbol{X}}\frac{d\boldsymbol{X}}{d\boldsymbol{D}}$$

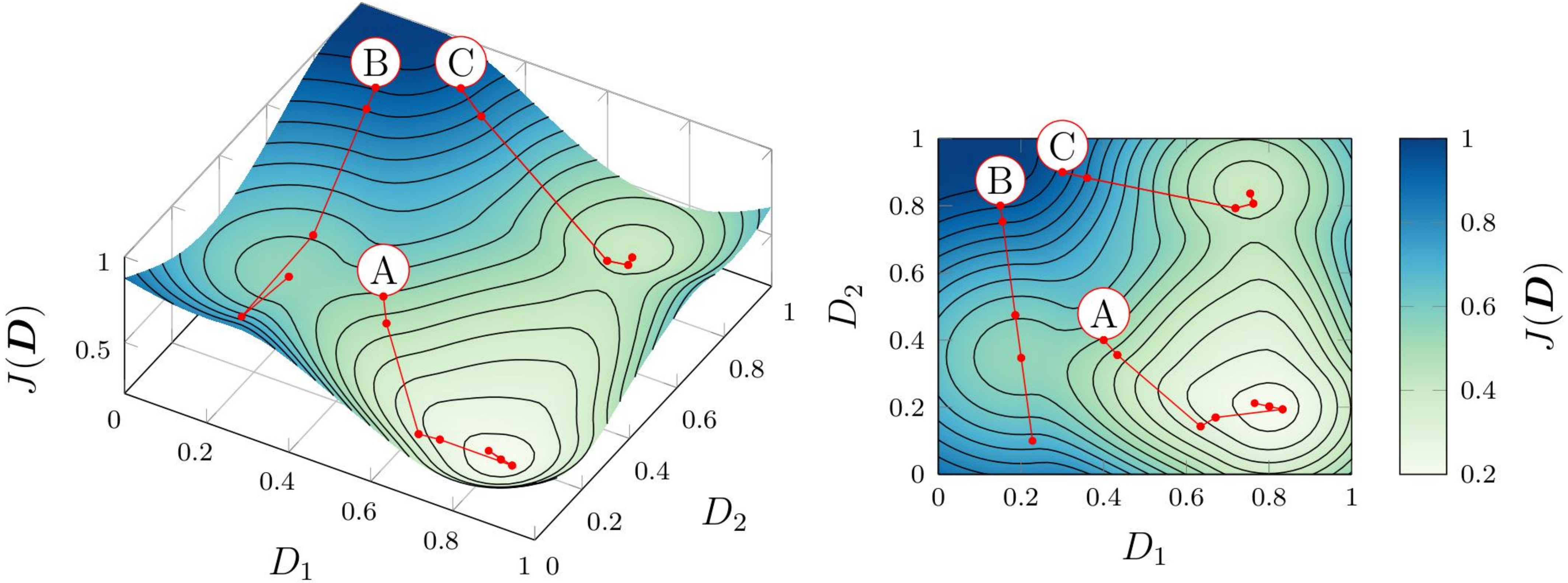
Adjoint equations:

$$\left[\frac{\partial \boldsymbol{R}}{\partial \boldsymbol{U}}\right]^T \boldsymbol{\lambda} = -\left[\frac{\partial f}{\partial \boldsymbol{U}}\right]^T$$

Only requires 1 flow solution & 1 adjoint solution for N design vars

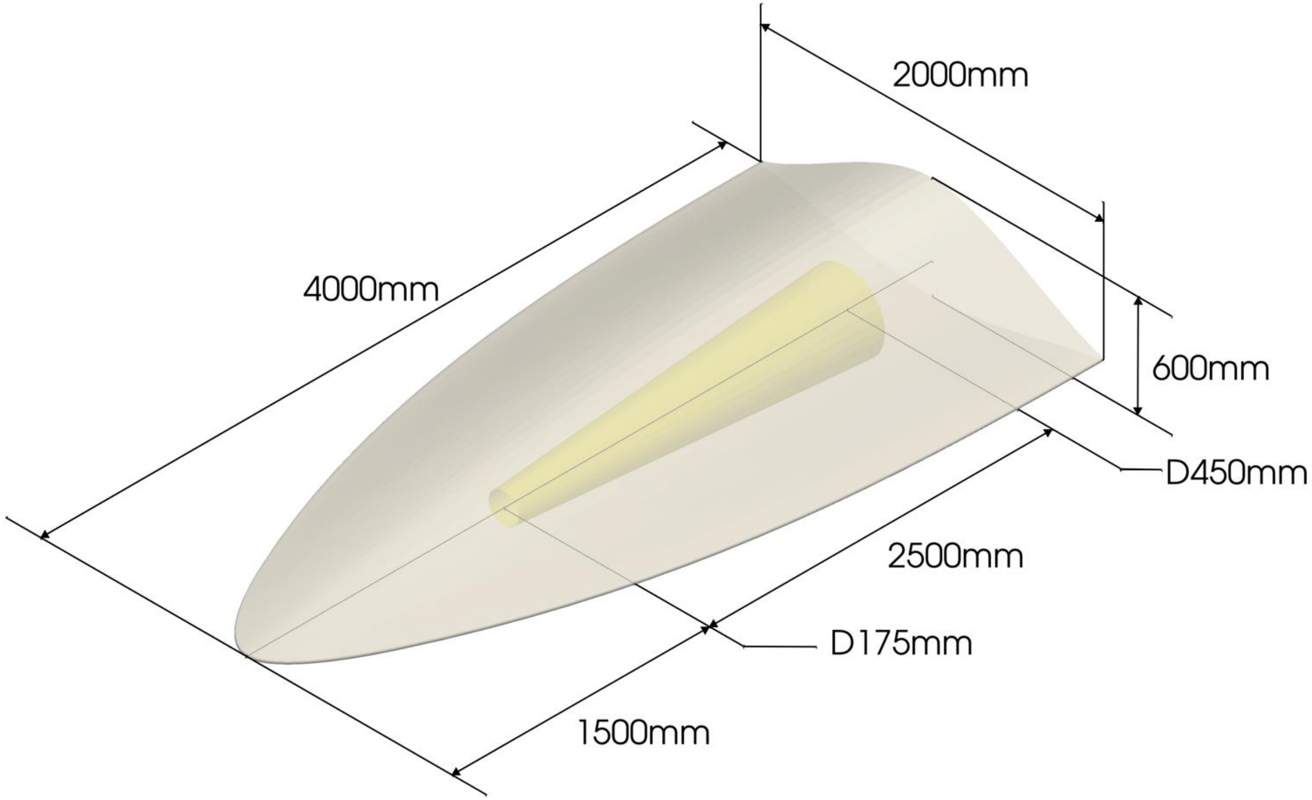
ASO Methodology – Optimiser

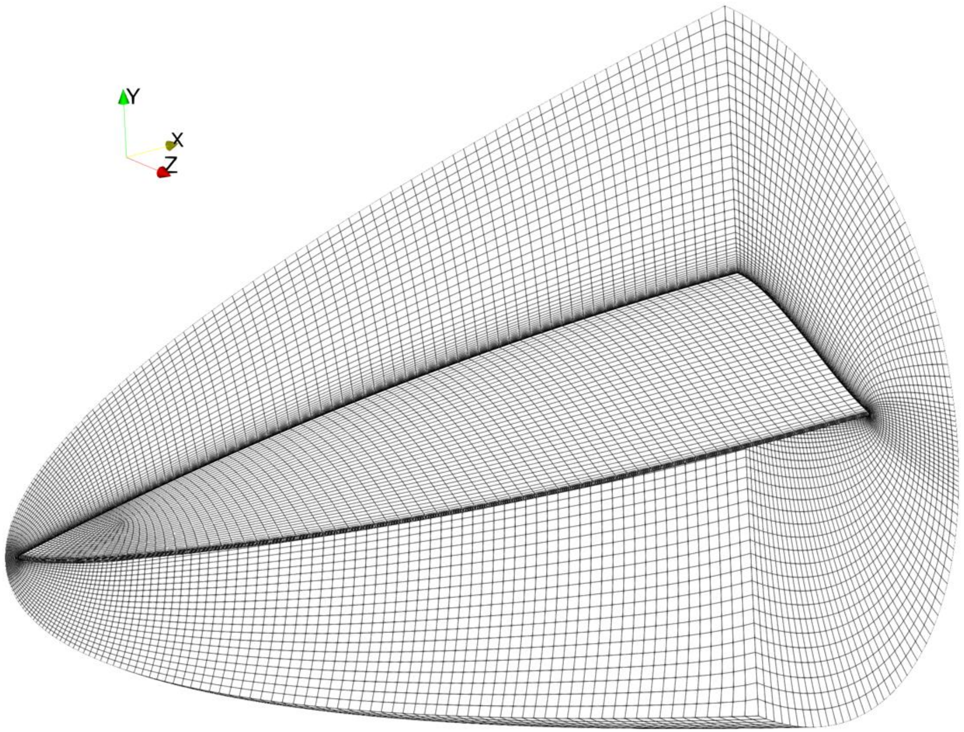




Sparse Nonlinear OPTimizer (SNOPT)

- Large-scale, constrained, non-linear optimisation problems
- Sequential Quadratic Programming (SQP) – gradient-based
- Quasi-Newton





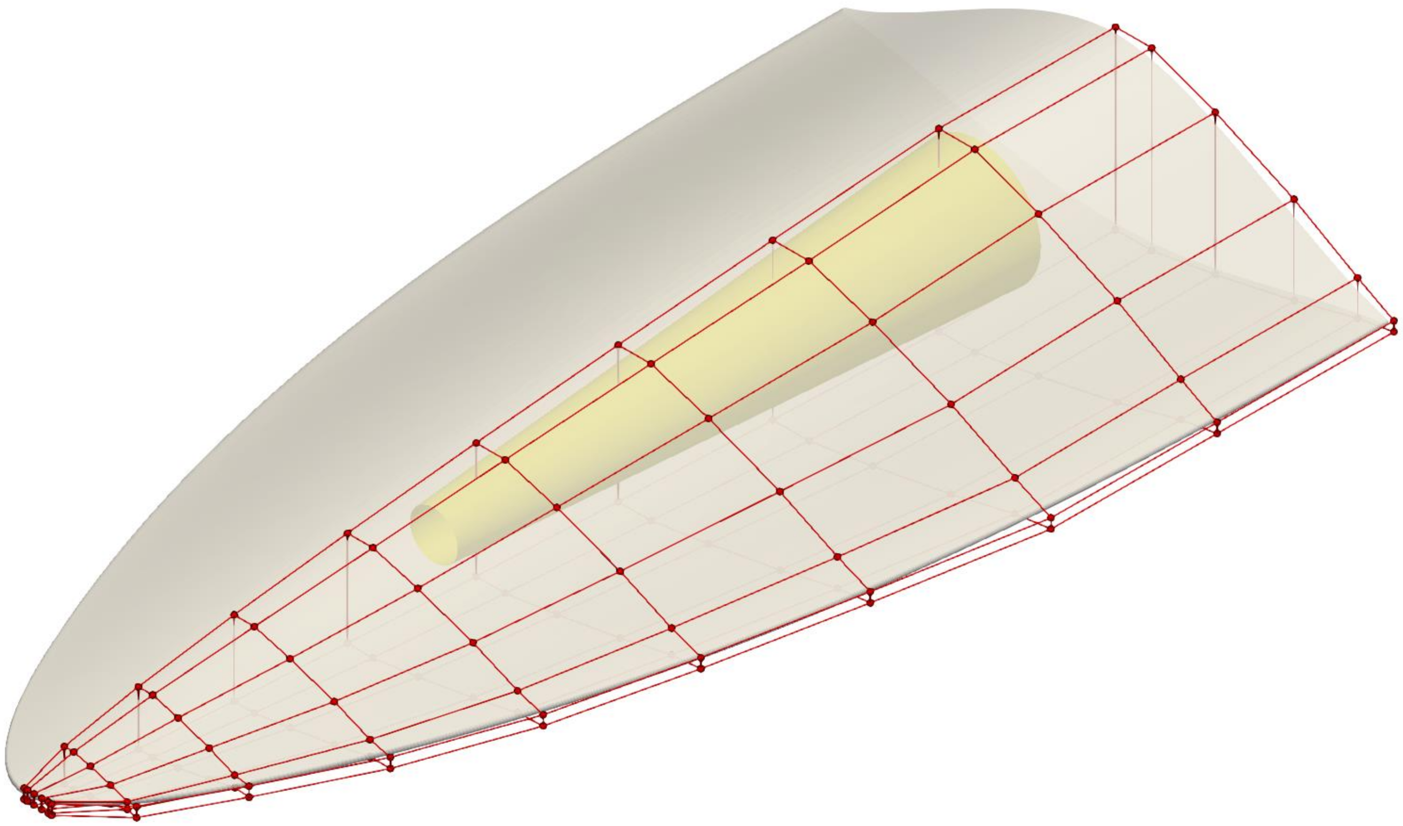
Optimisation problem

maximise $J = L/D$

subject to $V \geq V_{\text{payload}}$

- **Design point**: $M_\infty = 8$, $h = 40$km, $\alpha = 8°$
- **Modelling**: RANS w/ S-A turbulence
- **Gas model**: ideal air
- **Grid**: 660k cells

Lifting Body Optimisation – Shape Parameterisation

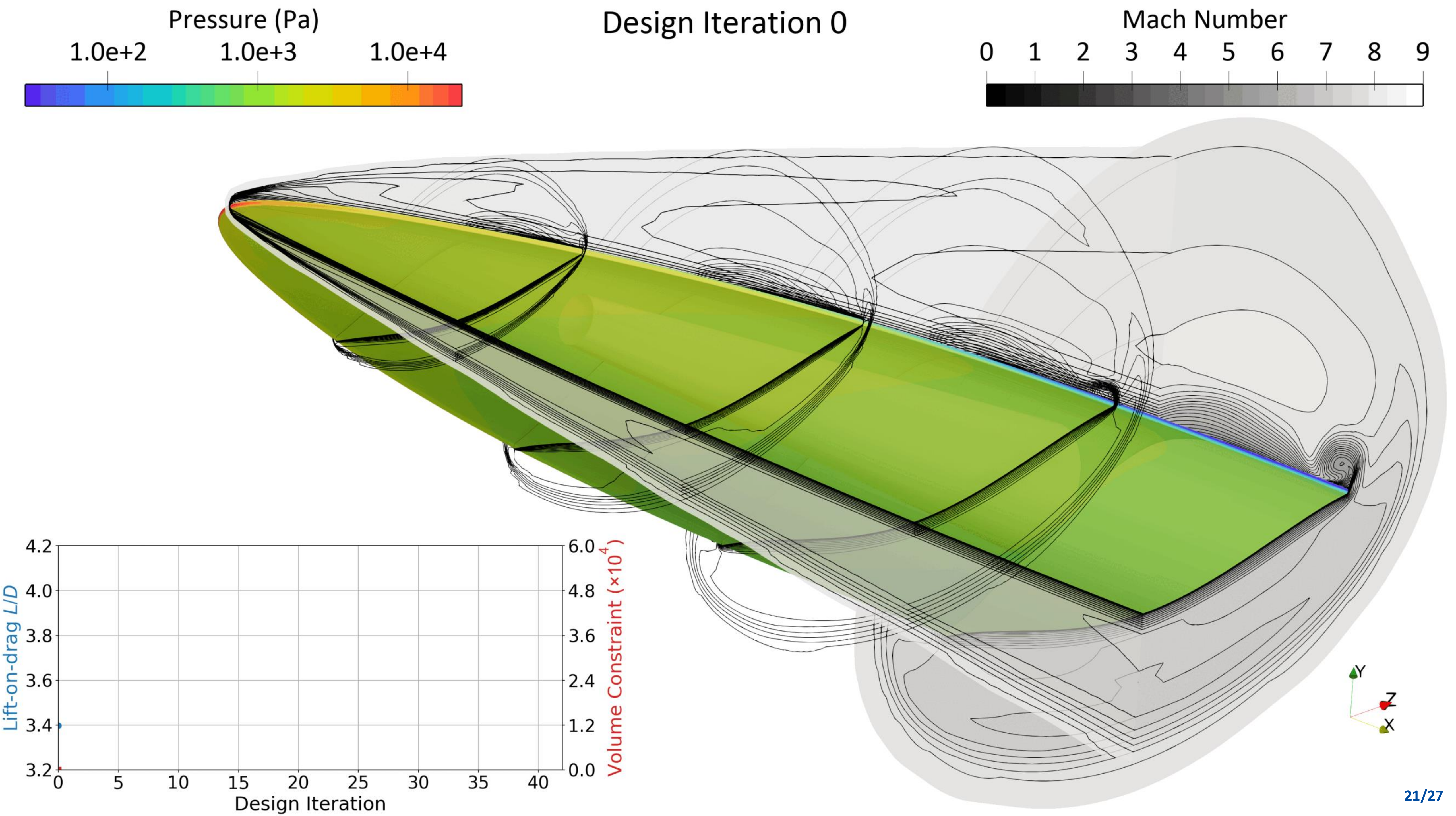


FFD hull
- 10 x 6 x 2 control points
- Fixed length

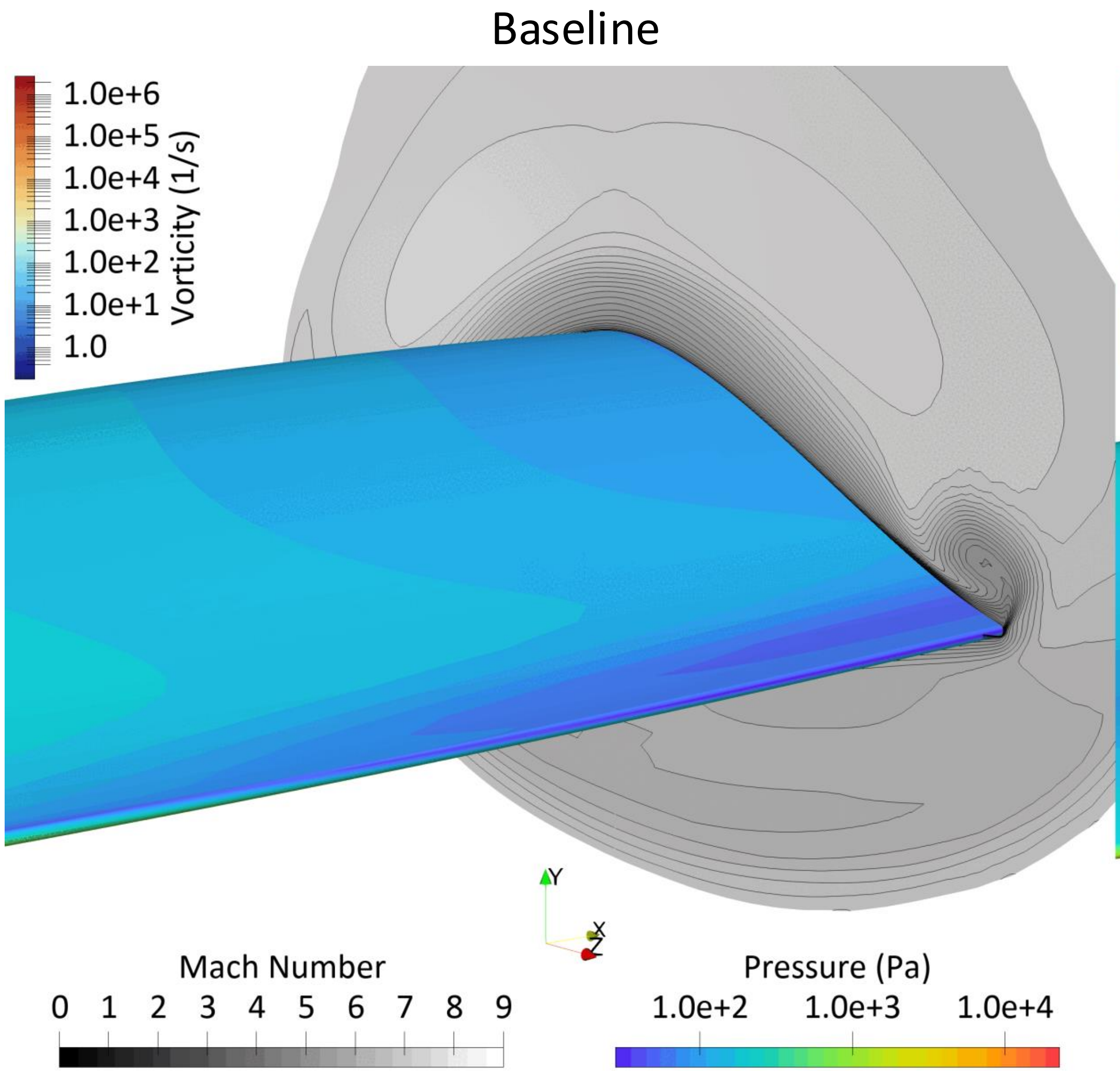
Surface DVs (118 total)
- FFD planes
  - Axial translation & width scale
- Control point pairs
  - Vertical scale & translation

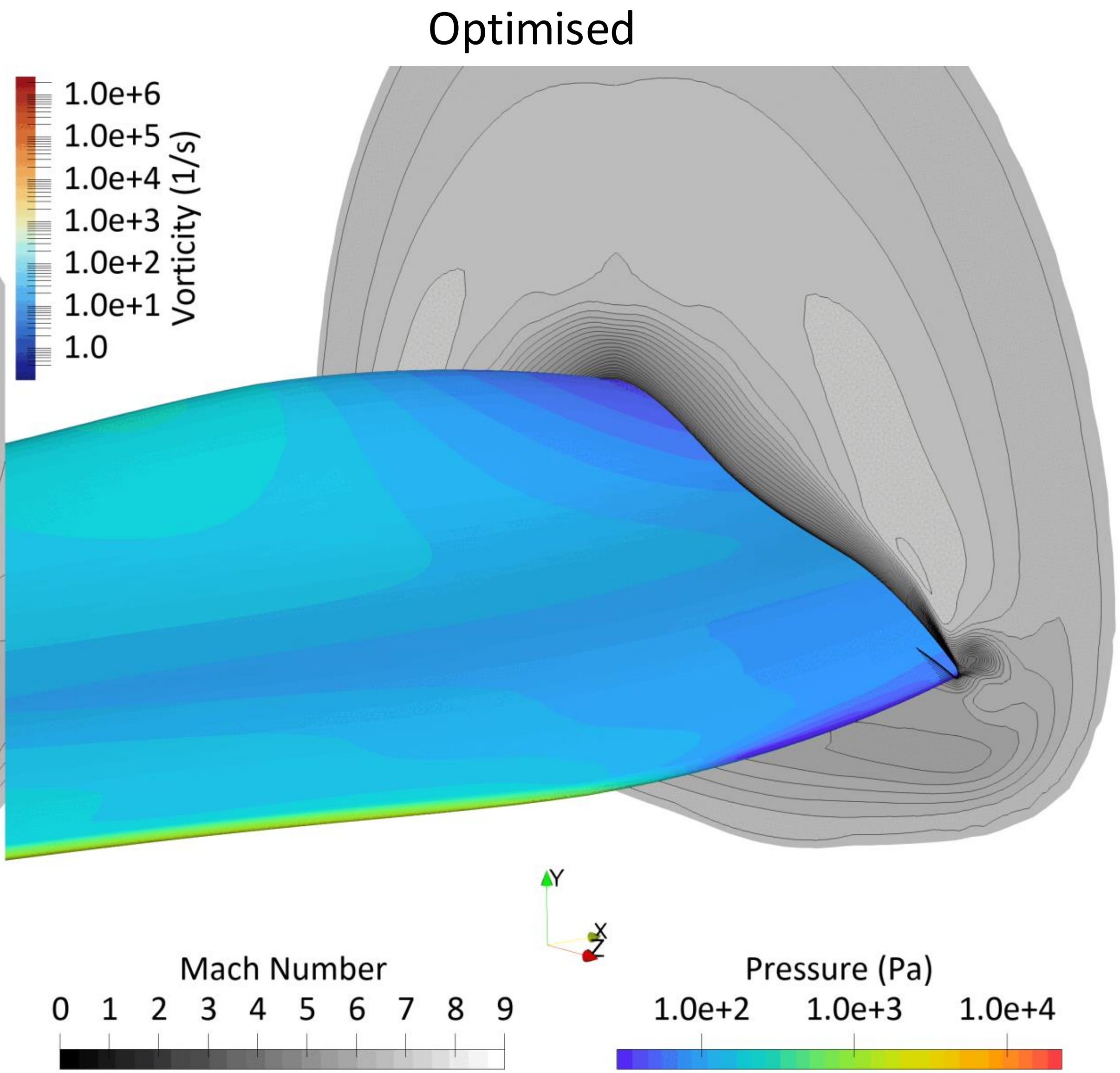
Payload DVs (3 total)
- Axial translation
- Vertical translation
- Pitching rotation

121 design variables overall

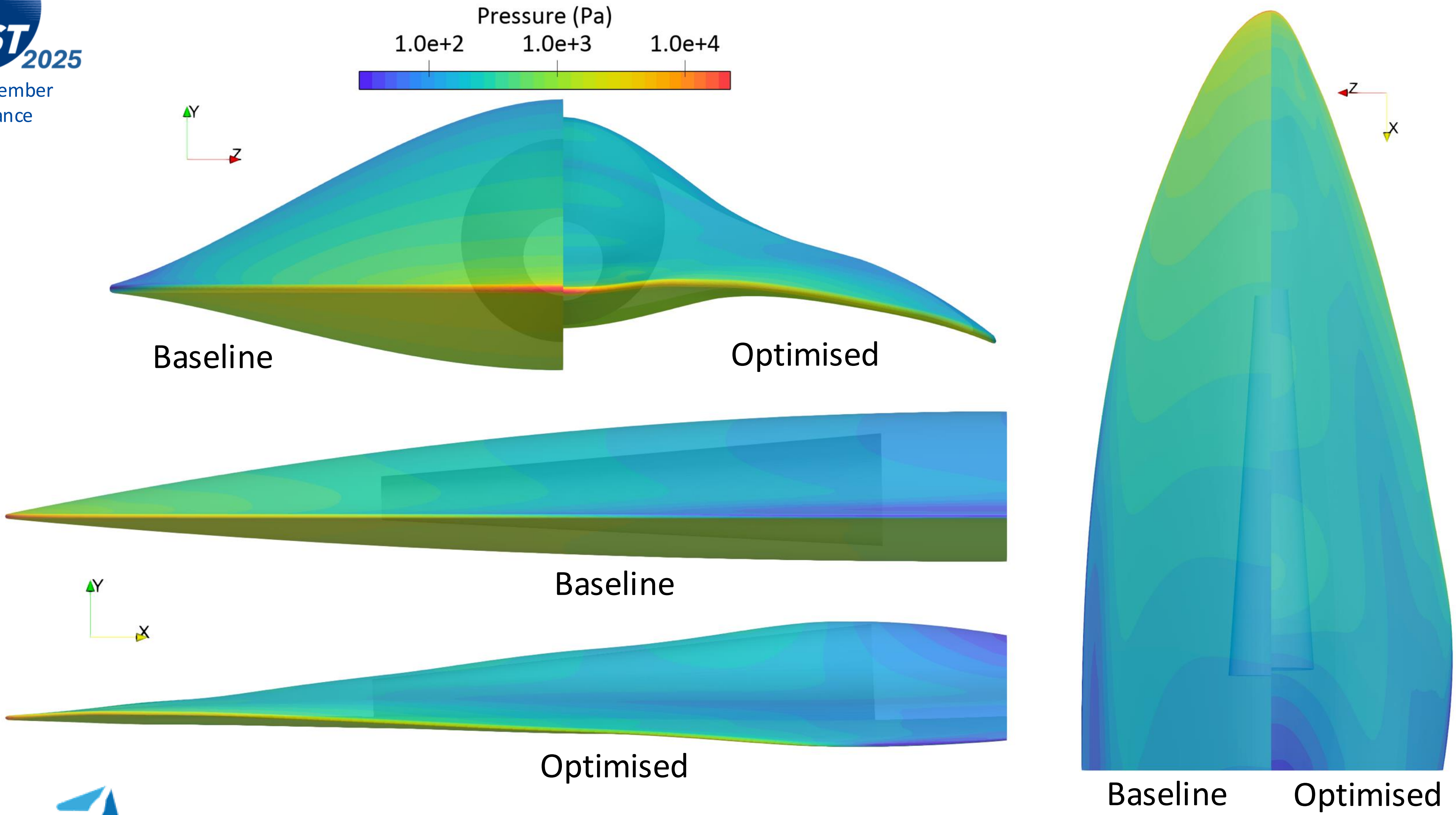

Pressure (Pa)
1.0e+2
1.0e+3
1.0e+4
Design Iteration 0
Mach Number
0 1 2 3 4 5 6 7 8 9
Lift-on-drag L/D
Volume Constraint (×10^4)
4.2
4.0
3.8
3.6
3.4
3.2
6.0
4.8
3.6
2.4
1.2
0.0
0 5 10 15 20 25 30 35 40
Design Iteration
Y
Z
X

Lifting Body Optimisation – Side Vortex Reduction

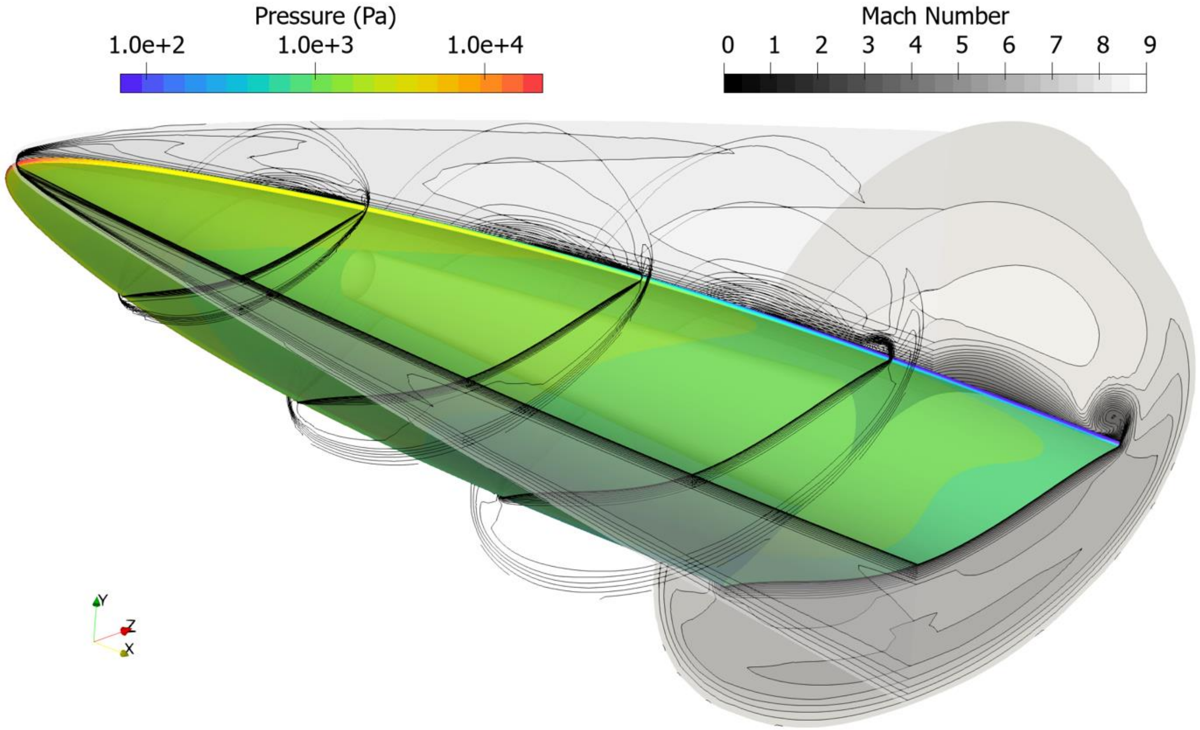




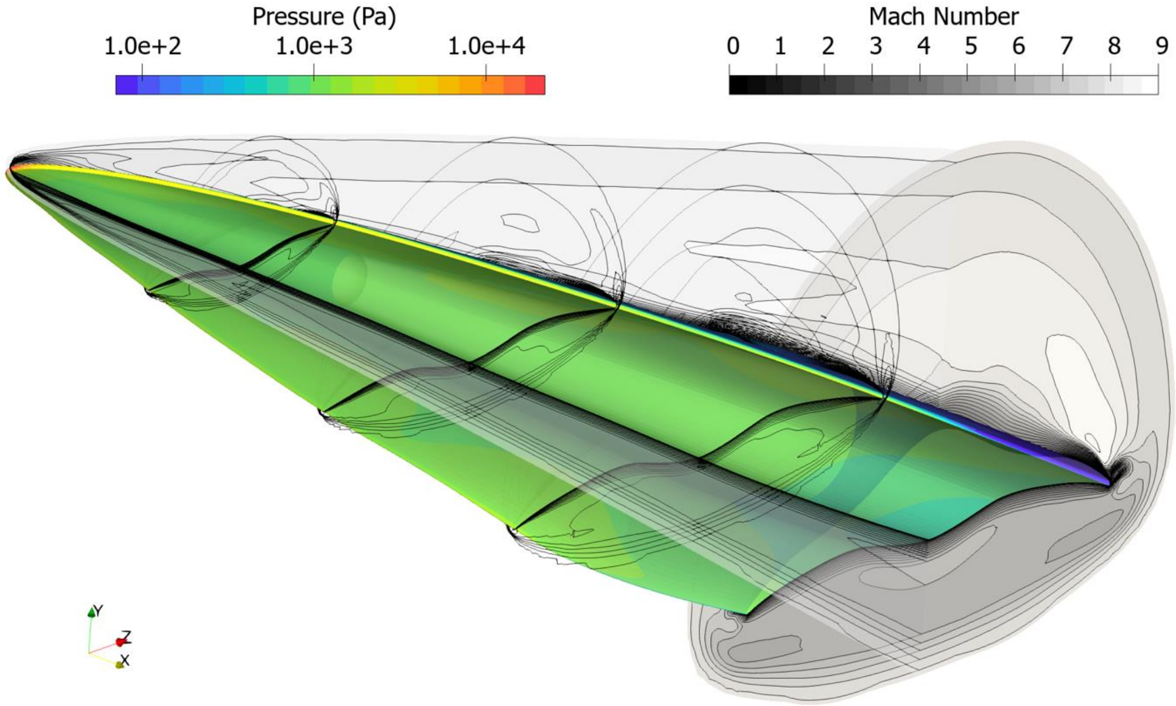
Lifting Body Optimisation – Shape Comparison



Lifting Body Optimisation – Performance Comparison



Baseline



Optimised

Configuration	Drag D (N)	Lift L (N)	Lift-on-drag L/D
Baseline	939	3190	3.40
Optimised	626	2589	4.14
Difference (%)	-33.3	-18.8	+21.8

Lifting Body Optimisation – Computational Costs

Optimisation case was executed on a workstation

- Dual Intel Xeon Silver 4216 CPUs @ 2.10 GHz (32 cores total)
- 196 GB RAM

Breakdown of run time

- 43 design iterations
- Average CFD solution: 45 mins
- Average Jacobian calculation: 52 mins
- Average adjoint solution: 14 mins
- Total run time: **4 days**

If we used finite differences instead of the adjoint method…

Time = (43 iterations) x (45 mins/CFD sol) x (121 + 1 DVs) = **164 days**

Conclusions and Future Work

Developed ASO framework for 3D hypersonic configurations

- High-fidelity CFD
- Adjoint-based sensitivity analysis
- Two-level FFD shape parameterisation

Performed aerodynamic optimisation of a hypersonic lifting body

- 21.8% improvement in L/D, while satisfying volume constraint
- A speed up of 41x using adjoint method, compared to FD

Future work

- Extend ASO modelling compatibility – reacting, multi-temperature
- Explore other design applications – shape-transition inlets/nozzles

Thank you for listening

Any questions?